WORK → Conceptually, its something you can calculate when a force (F) is acting on something that moves a distance (d).

The simplified definition is

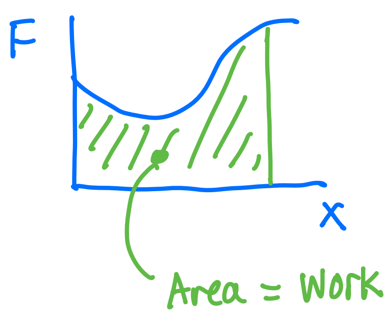
$$W = Fd^{*}$$ → So units are Nm

There are 2 assumptions with that, so we will make a better definition in a bit.

① F is constant.

* ② F is in the same direction as d.

① So what if the force is not constant? It turns out to be pretty simple, as long as you know calculus. ◡̈

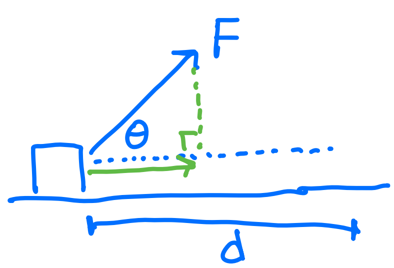


Let's imagine the force varies depending on where it is, like in the picture to the left for example.

The work would just be the Area under the graph, which means we would say

$$W = \int F dx$$

② What about the directions?

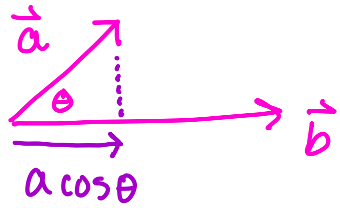


If F is not parallel to d, we need the component of F in the direction of d. So the green arrow is $F\cos\theta$ and is the part of F that is in the direction of d.

A non-calculus physics book might then say $W = Fd\cos\theta$

A math book might say we just multiplied the projection of the force vector onto the displacement vector.

Hey! That is a dot product of two vectors!



$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

[$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$ is another way of doing it]

Putting all this together is our <u>real</u> definition of Work.

$$W = \int_i^f \vec{F}\cdot d\vec{x}$$

Other Comments:

a) Dot products are also called scalar products

Work is a <u>scalar</u> quantity

b) Work can be negative! Notice if $90° < \theta < 180°$ that $\cos\theta$ is negative. If the force is in the opposite direction of the displacement the work done by that force is negative.

c) Work can be 0! If $\theta = 90°$, there is no part of the force in the direction of the displacement (and $\cos 90° = 0$) so it does no work

d) Since there are usually multiple forces acting on an object, you can add up the work done by each force to get the net work done on the object — or you could just figure out the work done by the net force.